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Stability Analysis of Mosquito Life Span Model with Delay

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Abstract

In this article, we have incorporated the time delay from mosquitoes searching for oviposition sites into searching for hosts into the nonlinear system of equations [1]. The effect of delay on the stability of the persistent equilibrium and Hopf bifurcation has been investigated. Finally, Numerical simulations have been executed.

Keywords: delay differential equation, stability analysis.

1. Introduction

Mosquitoes are very common fragile insects with an adult life span that lasts about two weeks. A majority of mosquitoes end their life cycle as food for birds, dragonflies and spiders or are killed by the effects of nature such as wind, rain or drought. Blood is the only crucial for the development of mosquito eggs which require certain proteins found in the blood. Mosquitoes are vectors for many of the most important human infections. They can carry malaria, yellow fever, dengue fever and more. Malaria risk is highest in the vicinity of water where mosquitoes oviposit [8].

Mathematical models have been developed to analyze the malaria transmission between humans and mosquitoes with non-linear forces of infection in form of saturated incidence rates [13], Malaria model with stage structured mosquitoes [12], spread of malaria through sensitivity analysis [5], population dynamics with temperature and age dependent survival [3], breaking the life cycle of a mosquito that incorporate a time delay at the larva stage that accounts for the period of growth and development to pupa [2], estimation of seasonal variation of mosquito population [11], mosquito dispersal in a hetrogeneous environment [1].

Stability of a mathematical model of malaria transmission [10], a delayed Ross-Macdonald model for malaria transmission [6], mosquito dispersal in a hetrogeneous environment [1], predator-prey populations subjected to constant effort of harvesting [9], a ratio dependent predator-prey model with quadratic harvesting [14], HIV model [7], HIV primary infection [15] have been discussed. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission [16] has been detailed. Stability and bifurcation of a two-species Leslie-Gower predator-prey system with time delay [4] has been detailed.

Let us consider the following mathematical model to investigate the impact of dispersal and heterogeneous distribution of resources on the distribution and dynamics of mosquito populations [1].

$$\frac{dE}{dT} = b\rho_{A_o}A_o - (\mu_E + \rho_E)E \tag{1.1}$$

$$\frac{dL}{dT} = \rho_E E - (\mu_{L_1} + \mu_{L_2}L + \rho_L)L \tag{1.2}$$

$$\frac{dP}{dT} = \rho_L L - (\mu_P + \rho_P)P \tag{1.3}$$

$$\frac{dA_h}{dt} = \rho_P P + \rho_{A_o}A_o - (\mu_{A_h} + \rho_{A_h})A_h \tag{1.4}$$

$$\frac{dA_r}{dt} = \rho_{A_h}A_h - (\mu_{A_r} + \rho_{A_r})A_r \tag{1.5}$$

$$\frac{dA_o}{dt} = \rho_{A_r}A_r - (\mu_{A_o} + \rho_{A_o})A_o \tag{1.6}$$

where,
b - number of female eggs laid per oviposition.
ρ_E - egg hatching rate into larvae (day^{-1}).
ρ_L - rate at which larvae developing into pupae (day^{-1}).
ρ_P - rate at which pupae develope into adult or emergence rate (day^{-1}).
μ_E - egg mortality rate (day^{-1}).
μ_P - pupae mortality rate (day^{-1}).
μ_{L_1} - density independent larvae mortality rate (day^{-1}).
μ_{L_2} - density dependent larvae mortality rate (day^{-1}).
ρ_{A_h} - rate at which host-seeking mosquitoes enter the resting state (day^{-1}).
ρ_{A_r} - rate at which resting mosquitoes enter oviposition site searching state (day^{-1}).

ρ_{A_o} - oviposition rate (day^{-1}).
μ_{A_h} - mortality rate of mosquitoes of searching for hosts (day^{-1}).
μ_{A_r} - mortality rate of resting mosquitoes (day^{-1}).
μ_{A_o} - mortality rate of mosquito searching for oviposition sites (day^{-1}).

The population reproduction number R_0 is

$$\frac{b\Pi_j(\frac{\rho_j}{\mu_j+\rho_j})}{1-\Pi_{A_i}(\frac{\rho_{A_j}}{\mu_{A_i}+\rho_{A_i}})}$$

where $j = E, L, P, A_h, A_r, A_0$ and $i = h, r$ and o.

We have incorporated the time delay (τ) from mosquitoes searching for oviposition sites into searching for hosts into the above model. Thus, we have the following delay model:

$$E' = b\rho_{A_o}A_o - (\mu_E + \rho_E)E \tag{1.7}$$
$$L' = \rho_E E - (\mu_{L_1} + \mu_{L_2}L + \rho_L)L \tag{1.8}$$
$$P' = \rho_L L - (\mu_P + \rho_P)P \tag{1.9}$$
$$A_h{}' = \rho_P P + \rho_{A_o}A_o(t-\tau) - (\mu_{A_h} + \rho_{A_h})A_h \tag{1.10}$$
$$A_r{}' = \rho_{A_h}A_h - (\mu_{A_r} + \rho_{A_r})A_r \tag{1.11}$$
$$A_o{}' = \rho_{A_r}A_r - \mu_{A_o}A_o - \rho_{A_o}A_o(t-\tau) \tag{1.12}$$

In section 2, stability analysis of the system of equations (1.7)–(1.12) have been discussed at persistent equilibrium
$P_e = (E^*, L^*, P^*, A_h{}^*, A_r{}^*, A_o{}^*)$,
where
$E^* = \dfrac{b\rho_{A_o}A_o{}^*}{\mu_E + \rho_E}$,
$L^* = \dfrac{(\mu L_1 + \rho_L)(R_0 - 1)}{\mu_{L_2}}$,
$P^* = \dfrac{\rho_L L^*}{\mu_P + \rho_P}$,
$A_h{}^* = \dfrac{\rho_P P^* R_0}{(\mu_{A_h} + \rho_{A_h})B_1}$,
$A_r{}^* = \dfrac{\rho_{A_h}A_h{}^*}{\mu_{A_r} + \rho_{A_r}}$,
$A_o{}^* = \dfrac{\rho_{A_r}A_r{}^*}{\mu_{A_o} + \rho_{A_o}}$
where

$$R_0 = \frac{B_1}{1-\prod_{A_i}\left(\frac{\rho_{A_i}}{\mu_{A_i}+\rho_{A_i}}\right)},$$

$$B_1 = b \prod_j \left(\frac{\rho_j}{\mu_j + \rho_j} \right),$$

$j = E, L, P, A_h, A_r, A_o$ and $i = A_h, A_r, A_o$.
In section 3, numerical simulations have been executed.

2. Stability analysis

Theorem 2.1. If $R_0 < 1$, then the mosquito free equilibrium point of system (1.7)-(1.12) is locally asymptotically stable for all $\tau \geq 0$.

Now we investigate the effect of the time delay on the stability of the persistent equilibrium P_e. The required jacobian matrix at P_e is given by,

$$J_{P_e} = \begin{pmatrix} -(\mu_E + \rho_E) & 0 & 0 & 0 & 0 & b\rho_{A_o} \\ \rho_E & -(\mu_{L_1} + \rho_L) - \phi & 0 & 0 & 0 & 0 \\ 0 & \rho_L & -(\mu_P + \rho_P) & 0 & 0 & 0 \\ 0 & 0 & \rho_P & -(\mu_{A_h} + \rho_{A_h}) & 0 & \rho_{A_o} e^{-\lambda\tau} \\ 0 & 0 & 0 & \rho_{A_h} & -(\mu_{A_r} + \rho_{A_r}) & 0 \\ 0 & 0 & 0 & 0 & \rho_{A_r} & -(\mu_{A_o} + \rho_{A_o} e^{-\lambda\tau}) \end{pmatrix}$$

where, $\phi = 2(\mu_{L_1} + \rho_L)(R_0 - 1)$.

To evaluate the eigenvalues of J_{P_e}, we solve $det(J_{P_e} - \lambda I) = 0$. We use the concept of block matrices to obtain this determinant.
Let $J = J_{P_e} - \lambda I$ be a block matrix given by,

$$J = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

with the following components:

$$A = \begin{pmatrix} -(\mu_E + \rho_E) - \lambda & 0 & 0 \\ \rho_E & -(\mu_{L_1} + \rho_L) - \phi - \lambda & 0 \\ 0 & \rho_L & -(\mu_P + \rho_P) - \lambda \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & b\rho_{A_o} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & \rho_P \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and,

$$D = \begin{pmatrix} -(\mu_{A_h} + \rho_{A_h}) - \lambda & 0 & \rho_{A_o} e^{-\lambda\tau} \\ \rho_{A_h} & -(\mu_{A_r} + \rho_{A_r}) - \lambda & 0 \\ 0 & \rho_{A_r} & -(\mu_{A_o} + \rho_{A_o} e^{-\lambda\tau}) - \lambda \end{pmatrix}$$

Therefore the concepts of block matrices that $det(J) = det(AD - BC)$. Since BC is a zero matrix, then $det(J) = det(AD) = 0$. Then, the characteristic equation is,

$$\Big[\lambda^6 + \lambda^5(A + a + b) + \lambda^4(B + A(a + b) + ab + c) + \lambda^3(C + (a + b)B + A(ab + c) + ac) + \lambda^2(C(a + b) + B(ab + c) + Aac) + \lambda(C(ab + c) + Bac) + acC\Big] + \Big[\lambda^5 D + \lambda^4(E + (a + b)D) + \lambda^3(F + (a + b)E + D(ab + c)) + \lambda^2((a + b)F + E(ab + c) + acD) + \lambda(F(ab + c) + Eac) + acF\Big]e^{-\lambda\tau} = 0$$

where,

$$\begin{aligned}
&a = \mu_E + \rho_E, \quad b = \mu_{L_1} + \rho_L + \phi + \mu_P + \rho_P,\\
&c = (\mu_{L_1} + \rho_L + \phi)(\mu_P + \rho_P),\\
&A = \mu_{A_h} + \rho_{A_h} + \mu_{A_r} + \rho_{A_r} + \mu_{A_o},\\
&B = (\mu_{A_r} + \rho_{A_r})(\mu_{A_h} + \rho_{A_h}) + \mu_{A_o}(\mu_{A_r} + \rho_{A_r} + \rho_{A_h} + \mu_{A_h}),\\
&C = \mu_{A_o}(\mu_{A_r} + \rho_{A_r})(\mu_{A_h} + \rho_{A_h}),\\
&D = \rho_{A_o}, \quad E = \rho_{A_o}(\mu_{A_h} + \rho_{A_h} + \mu_{A_r} + \rho_{A_r}),\\
&F = \rho_{A_o}(\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r}) - \rho_{A_o}\rho_{A_h}\rho_{A_r}.
\end{aligned}$$

Then,

$$P(\lambda) + Q(\lambda)e^{-\lambda\tau} = 0 \tag{2.1}$$

where

$$\begin{aligned}
P(\lambda) &= \lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6\\
Q(\lambda) &= b_1\lambda^5 + b_2\lambda^4 + b_3\lambda^3 + b_4\lambda^2 + b_5\lambda + b_6
\end{aligned}$$

where

$$\begin{aligned}
&a_1 = A + (a + b), \quad a_2 = B + A(a + b) + ab + c,\\
&a_3 = C + (a + b)B + A(ab + c) + ac, \quad a_4 = C(a + b) + B(ab + c) + Aac,\\
&a_5 = C(ab + c) + Bac, \quad a_6 = acC, \quad b_1 = D,\\
&b_2 = E + (a + b)D, \quad b_3 = F + (a + b)E + D(ab + c),\\
&b_4 = (a + b)F + E(ab + c) + acD, \quad b_5 = F(ab + c) + Eac, \quad b_6 = acF.
\end{aligned}$$

If $\tau = 0$, then the above characteristic equation becomes,

$$\lambda^6 + (a_1 + b_1)\lambda^5 + (a_2 + b_2)\lambda^4 + (a_3 + b_3)\lambda^3 + (a_4 + b_4)\lambda^2 + (a_5 + b_5)\lambda + (a_6 + b_6) = 0$$

Theorem 2.2. If $R_0 > 1$ and $p > 0$, $pq > r$, $r(pq - r) > p(ps - t)$, and $(ps - t)[(pq - r)r - p(ps - t)] > (pq - r)[(pq - r)t - up^2]$, then the persistent equilibrium P_e is locally asymptotically stable for all delay $\tau \geq 0$.

Proof. If $R_0 > 1$ and $\tau > 0$, assuming $\lambda = i\omega$ with $\omega > 0$ in (2.1) we get,

$$\begin{aligned}
&- \omega^6 + ia_1\omega^5 + a_2\omega^4 - ia_3\omega^3 - a_4\omega^2 + ia_5\omega + a_6 \\
&\quad + [ib_1\omega^5 + b_2\omega^4 - ib_3\omega^3 - b_4\omega^2 \\
&\quad + ib_5\omega + b_6](cos\omega\tau - isin\omega\tau) = 0
\end{aligned}$$

Separating the real and imaginary parts, we get,

$$\begin{aligned}
-\omega^6 + a_2\omega^4 - a_4\omega^2 + a_6 &= (-b_2\omega^4 + b_4\omega^2 - b_6)cos\omega\tau \\
&\quad + (-b_1\omega^5 + b_3\omega^3 - b_5\omega)sin\omega\tau \qquad (2.2) \\
a_1\omega^5 - a_3\omega^3 + a_5\omega &= (-b_1\omega^5 + b_3\omega^3 - b_5\omega)cos\omega\tau \\
&\quad - (-b_2\omega^4 + b_4\omega^2 - b_6)sin\omega\tau \qquad (2.3)
\end{aligned}$$

Squaring and adding (2.2) and (2.3),

$$\begin{aligned}
&\omega^{12} + \omega^{10}({a_1}^2 - 2a_2 - {b_1}^2) + \omega^8({a_2}^2 + 2(a_4 - a_1a_3 + b_1b_3) - {b_2}^2) + \omega^6({a_3}^2 \\
&\quad + 2(a_1a_5 - a_6 - a_2a_4 - b_1b_5 + b_2b_4) - {b_3}^2) + \omega^4({a_4}^2 + 2(a_2a_6 - a_3a_5 - b_2b_6 \\
&\quad + b_3b_5) - {b_4}^2) + \omega^2({a_5}^2 + 2(b_4b_5 - a_4a_6) - {b_5}^2) + ({a_6}^2 - {b_6}^2) = 0 \qquad (2.4)
\end{aligned}$$

Let

$$\begin{aligned}
&z = \omega^2, \quad p = {a_1}^2 - 2a_2 - {b_1}^2, \\
&q = {a_2}^2 + 2(a_4 - a_1a_3 + b_1b_3) - {b_2}^2, \\
&r = {a_3}^2 + 2(a_1a_5 - a_6 - a_2a_4 - b_1b_5 + b_2b_4) - {b_3}^2, \\
&s = {a_4}^2 + 2(a_2a_6 - a_3a_5 - b_2b_6 + b_3b_5) - {b_4}^2, \\
&t = {a_5}^2 + 2(b_4b_5 - a_4a_6) - {b_5}^2
\end{aligned}$$

and $u = {a_6}^2 - {b_6}^2$. Thus, we have

$$G(z) \quad = \quad z^6 + pz^5 + qz^4 + rz^3 + sz^2 + tz + u = 0 \qquad (2.5)$$

Since $p > 0$, $pq > r$, $r(pq - r) > p(ps - t)$ and $(ps - t)[(pq - r)r - p(ps - t)] > (pq - r)[(pq - r)t - up^2]$, then $z = \omega^2 < 0$. So our assumption that $\lambda = i\omega$ is a root of (2.1) is wrong which means that (2.1) has no positive roots and the real parts of all eigenvalues are negative for all delay $\tau \geq 0$.

3. Hopf Bifurcation

If $u < 0$, then $G(0) = u < 0$ and $lim_{z \to \infty} G(z) = \infty$. Then there exists atleast a positive root satisfying equation (2.1), so the charateristic equation (2.1) has at a pair of purely imaginary roots of the form $\pm i\omega_0$. Eliminating $sin\omega\tau$ from (2.2) and (2.3), we have

$$\cos\omega\tau = \frac{\begin{array}{c}[(-b_2\omega^4 + b_4\omega^2 - b_6)(-\omega^6 + a_2\omega^4 - a_4\omega^2 + a_6) \\ +(-b_1\omega^5 + b_3\omega^2 - b_5\omega)(a_1\omega^5 - a_3\omega^3 + a_5\omega)]\end{array}}{(-b_2\omega^4 + b_4\omega^2 - b_6)^2 + (-b_1\omega^5 + b_3\omega^3 - b_5\omega)^2}$$

Therefore, τ_n^* corresponding to ω_0 is given by;

$$\tau_n^* = \frac{1}{\omega_0} cos^{-1} \frac{\begin{array}{c}[(-b_2\omega_0^4 + b_4\omega_0^2 - b_6)(-\omega_0^6 + a_2\omega_0^4 - a_4\omega_0^2 + a_6) \\ +(-b_1\omega_0^5 + b_3\omega_0^2 - b_5\omega_0)(a_1\omega_0^5 - a_3\omega_0^3 + a_5\omega_0)]\end{array}}{(-b_2\omega_0^4 + b_4\omega_0^2 - b_6)^2 + (-b_1\omega_0^5 + b_3\omega_0^3 - b_5\omega_0)^2} + \frac{2n\pi}{\omega_0}$$

If $\tau = 0$, the persistent equilibrium P_e is stable when $R_0 > 1$ [1]. Therefore P_e is stable for $\tau < \tau_0$ where $\tau_0 = \tau_0^*$ as $n = 0$. Hence, if (2.1) has a pair of purely imaginary roots, then it has roots with positive real part (by continuity in τ). Thus, P_e becomes unstable and periodic solutions may happen that is Hopf bifurcation occur if $\frac{d(Re\lambda)}{d\tau}|_{\tau=\tau_0} > 0$.

Differentiating (2.1) with respect to τ, we get

$$\begin{aligned}&[(6\lambda^5 + 5a_1\lambda^4 + 4a_2\lambda^3 + 3a_3\lambda^2 + 2a_4\lambda + a_5) \\ &\quad + e^{-\lambda\tau}(5b_1\lambda^4 + 4b_2\lambda^3 + 3b_3\lambda^2 + 2b_4\lambda + b_5) \\ &\quad - \tau e^{-\lambda\tau}(b_1\lambda^5 + b_2\lambda^4 + b_3\lambda^3 + b_4\lambda^2 + b_5\lambda + b_6)]\frac{d\lambda}{d\tau} \\ &\qquad = \lambda e^{-\lambda\tau}(b_1\lambda^5 + b_2\lambda^4 + b_3\lambda^3 + b_4\lambda^2 + b_5\lambda + b_6)\end{aligned}$$

Then

$$\begin{aligned}\left(\frac{d\lambda}{d\tau}\right)^{-1} &= \frac{5\lambda^6 + 4a_1\lambda^5 + 3a_2\lambda^4 + 2a_3\lambda^3 + a_4\lambda^2 - a_6}{-\lambda^2(\lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6)} \\ &\quad + \frac{4b_1\lambda^5 + 3b_2\lambda^4 + 2b_3\lambda^3 + b_4\lambda^2 - b_6}{\lambda^2(b_1\lambda^5 + b_2\lambda^4 + b_3\lambda^3 + b_4\lambda^2 + b_5\lambda + b_6)} - \frac{\tau}{\lambda}\end{aligned}$$

Thus

$$
\begin{aligned}
&sign\left\{\frac{d(Re\lambda)}{d\tau}\right\}_{\lambda=i\omega_0}\\
&\quad= sign\left\{Re\left(\frac{d\lambda}{d\tau}\right)^{-1}\right\}_{\lambda=i\omega_0}\\
&\quad= sign\left(\frac{1}{\omega_0{}^2}\left[\frac{5\omega_0{}^{12}+4\omega_0{}^{10}(a_1^2-2a_2-b_1^2)+3\omega_0{}^8(a_2^2+2(a_4-a_1a_3+b_1b_3)-b_2^2)}{(b_2\omega_0{}^4-b_4\omega_0{}^2+b_6)^2+(b_1\omega_0{}^5-b_3\omega_0{}^3+b_5\omega_0)^2}\right.\right.\\
&\qquad+\frac{2\omega_0{}^6(a_3^2+2(a_1a_5-a_6-a_2a_4-b_1b_5+b_2b_4)-b_3^2}{(b_2\omega_0{}^4-b_4\omega_0{}^2+b_6)^2+(b_1\omega_0{}^5-b_3\omega_0{}^3+b_5\omega_0)^2}\\
&\qquad\left.\left.+\frac{\omega_0{}^4(a_4^2+2(a_2a_6-a_3a_5-b_2b_6+b_3b_5)-b_4^2))+(b_6^2-a_6^2)}{(b_2\omega_0{}^4-b_4\omega_0{}^2+b_6)^2+(b_1\omega_0{}^5-b_3\omega_0{}^3+b_5\omega_0)^2}\right]\right)
\end{aligned}
$$

If

$$
\begin{aligned}
(a_1^2-2a_2-b_1^2)&>0,\\
a_2^2+2(a_4-a_1a_3+b_1b_3)-b_2^2&>0,\\
a_3^2+2(a_1a_5-a_6-a_2a_4-b_1b_5+b_2b_4)-b_3^2&>0,\\
a_4^2+2(a_2a_6-a_3a_5-b_2b_6+b_3b_5)-b_4^2&>0,\\
b_6^2-a_6^2&>0
\end{aligned}
$$

which means that $p > 0, q > 0, r > 0, s > 0$ and $u < 0$, then

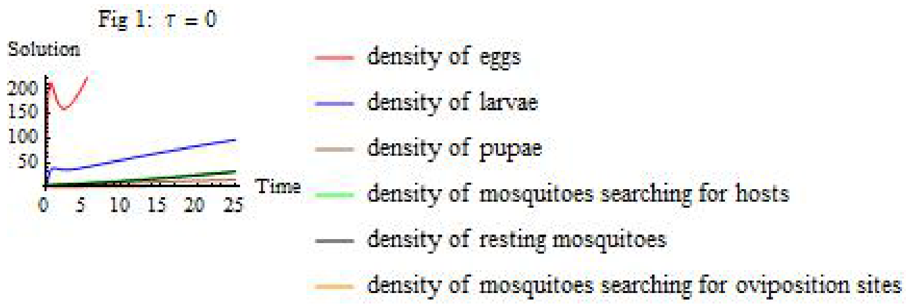
$$\frac{d(Re\lambda)}{d\tau}|_{\tau=\tau_0,\omega=\omega_0} > 0.$$

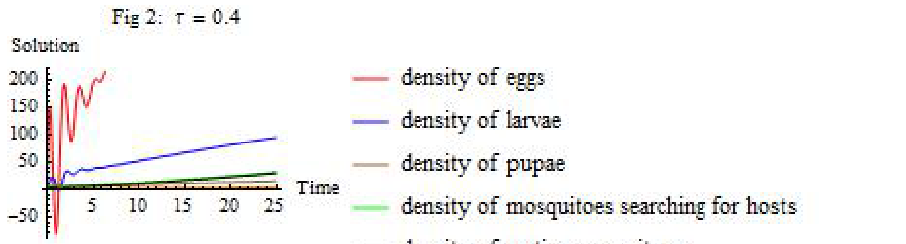
Hence we have atleast one eigenvalue with positive real part for $\tau > \tau_0$ and the conditions for Hopf bifurcation are satisfied yielding periodic solutions at $\tau = \tau_0, \omega = \omega_0$. Thus we have the following theorem:

Theorem 3.1. If $R_0 > 1$ and $p > 0, q > 0, r > 0, s > 0$ and $u < 0$, the persistent equilibrium P_e remains stable for $\tau < \tau_0$ and unstable when $\tau > \tau_0$, a Hopf bifurcation occurs as τ passes through τ_0.

4. Numerical Simulation

In this section we present numerical results of the system (1.7)-(1.12) to verify the analytical predictions obtained in previous section. Let us consider the system with the parameter values $b = 100$, $\rho_E = 0.50$, $\rho_L = 0.14$, $\rho_p = 0.50$, $\mu_E = 0.39$, $\mu_{L_1} = 0.44$, $\mu_{L_2} = 0.05$, μ_p=0.37, $\rho_{A_h} = 0.46$, $\rho_{A_r} = 0.43$, $\rho_{A_0} = 3.0$, $\mu_{A_h} = 0.18$, $\mu_{A_r} = 0.0043$ and $\mu_{A_0} = 0.41$. So the system (1.7)-(1.12) has a positive equilibrium P_e(1480.19, 116, 18.67, 39.15, 41.47, 5.23).





5. Conclusion

We investigated the effect of time delay and Hopf bifurcation in the mosquito life cycle model. It is observed that if $R_o < 1$ the mosquito free equilibrium point of system is locally asymptotically stable for all $\tau \geq 0$ and if $R_o > 1$ the persistent equilibrium is locally asymptotically stable for all $\tau \geq 0$.

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